β-spectroscopy

Aim
Investigate the energy-distribution of β-particles.

Apparatus
β-source (Y-90/Sr-90), G.M.Tube, signal-amplifier, counter-device, quarter-circular particle-orbit, electro-magnet (500 windings), direct current power supply, alternating current power supply, ammeter, Hall probe.

Theory
The energy of decay of β-particles is quantized (determined by differences of energy-levels in the radioactive nucleus). In contrast to α- and γ-decay the β-decay energy (Q) is distributed between two products – the kinetic energy of the β-particle (E_β) and the kinetic energy of the (anti)neutrino (E_ν):

$$Q = E_\beta + E_\nu$$

As a result of this, the kinetic energy of the β-particle (E_β) has a continuous distribution between 0 ($E_\beta = Q - E_\nu = 0$) and a maximum ($E_\beta = Q - 0$). This distribution and the value of Q is the goal of this investigation.

As the β-particles carry electrical charge (= -e as they are electrons) the kinetic energy E_β can be determined by deflection of the β-particles in a homogenous magnetic field B. The orbit is circular with radius r, and as the magnetic force must be equal to the centripetal force the speed can be determined classically by:

$$e\,v\,B = m\,v^2 / r$$

A calculation of v using typical values of B = 0.3 T, r = 0.03 m yields the unlikely result of a v > c! Find v! The reason is of course that we must make relativistic calculations.

The classical equation above can be changed to an equation of the momentum p:

$$p = m\,v = r\,e\,B$$

and it can be shown that these expressions (with m being the relativistic mass = $\gamma\, m_0$, m_0 the rest-mass of the electron) are still valid according to the theory of relativity.

The kinetic energy is then given by the relativistic expression:

$$E_\beta = E - m_0c^2 = \sqrt{p^2c^2 + m_0{}^2c^4} - m_0c^2$$

with p = r e B.

To avoid a lot of calculations it is more convenient to express energies in eV. The SI-unit of $p^2\,c^2 = r^2\,e^2\,B^2\,c^2$ is J^2, with e = 1.602 10^{-19} C. To change to $(eV)^2$ we must divide by the number $(1.602\ 10^{-19})^2 = e^2$. The rest-mass of the electron is 5.11 10^5 eV/c^2. So expressing the kinetic energy in eV (with r in m, B in T and c in m/s) we have[1]:

$$E_\beta = \sqrt{r^2B^2c^2 + (5.11\cdot10^5 eV)^2} - 5.11\cdot10^5 eV$$

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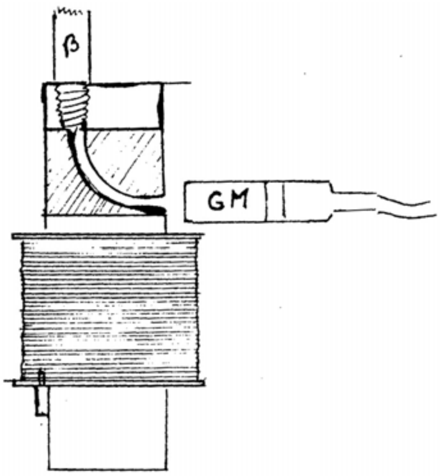
If you want a little challenge in relativistic calculations you can show that the speed is:

$$v = \frac{pc}{\sqrt{p^2 + m_0{}^2 c^2}}$$

which together with p = r e B makes it possible to calculate the relativistic correct speed v. Find v with the typical data given before.

The experiment
Assemble the quarter-circular orbit between the two coils of the electromagnet and connect the two coils in serial to the direct current power supply and the ammeter. Consider correct direction of current to bend negative charges in the circular track. Mount the Y-90/Sr-90 source (do *not* screw tight!) in the notched part of the quarter-circle. Position the GM-tube at the other end of the quarter-circle using suitable holders.



Make a series of measurements of counts N as a function of current I (0–3 A) through the electromagnet. Remember to measure background radiation counts for later corrections.

Calibrating the I–B dependence:
The relationship between I and B is not calculated here, but found experimentally using a Hall probe. Measure B as a function of I and plot the points in a graph to make it possible to read B for given I. (If using PC you can create a curve fitting to the plotted points, if not: just create the curve by hand on graph-paper).

Important advice:
The iron-core in the electromagnet "remembers" the magnet field – this phenomena called hysteresis. It is therefore important only to let the current *increase* during each measurement series of the experiment and of the Hall probe measurements, *never let the current decrease*. Besides this it is advisable to apply an alternating current to the electromagnet in 30 seconds before each measurement series is started up

The measured data together with the theory and the I-B dependence should lead to a plot showing counts as a function of the kinetic energy of the β-particles thus giving the energy-distribution (spectrum) of the β-particles. By suitable extrapolation of this distribution the maximum kinetic energy of β-particles (corresponding to Q) is found. Use an isotope map or table to compare with official values (both Sr-90 and Y-90 emits β-particles – which of them do you think is most important in this context?) Discuss random and systematic errors attempting to give some numerical judgements.

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[1] A more formal argument for this formula: Regard r, e, B and c as pure numbers without units, then p^2c^2 can be expressed as: $r^2e^2B^2c^2\ J^2 = r^2e^2B^2c^2\ C^2V^2 = r^2B^2c^2\ (eCV)^2$ where eCV correspond to the unit eV (e is the pure number $1.602\ 10^{-19}$).