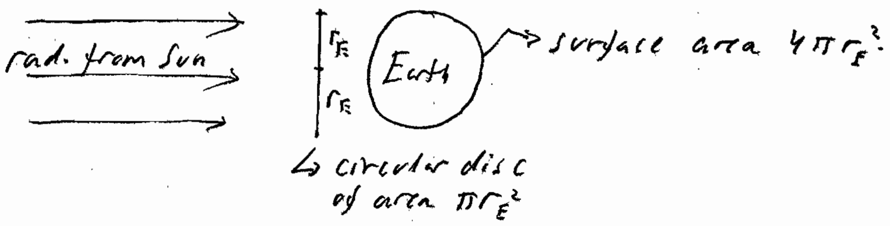
Radiation balance for Earth

Total radiation of Sun (luminosity of Sun): $L = 3.826 \times 10^{26}$ W.
Radiation intensity at Earth at distance $r_{SE} = 1.496 \times 10^{11}$ m:
Solar constant $S = \frac{L}{4\pi r_{SE}^2} = 1360\ \mathrm{W\,m^{-2}}$.

Total power hitting Earth, radius r_E: $S \cdot \pi r_E^2$
Average intensity over whole area of Earth:

$$\frac{S \pi r_E^2}{4\pi r_E^2} = \frac{1}{4} S = 340\ \mathrm{W\,m^{-2}}.$$



Power balance of Earth:

$$\text{Netto power in} = \text{power in} - \text{power out}$$
$$= S(1-\alpha)\pi r_E^2 - \varepsilon \sigma 4\pi r_E^2 T^4$$

α = albedo = 0.30. ε = emissivity ≈ 1.

Average intensity balance for Earth surface:
divide power balance with $4\pi r_E^2$:

*1 Netto intensity in $= I_{in} - I_{out}$

$$= \frac{1}{4} S(1-\alpha) - \varepsilon \sigma T^4.$$

Surface heat capacity $C_S \approx 4 \times 10^8\ \mathrm{J\,K^{-1}\,m^{-2}}$
Netto heat energy in $= Q = C_S A \Delta T$ (A area).

$$Q = (I_{in} - I_{out}) \cdot A \cdot \Delta t$$

*2 so: $\Delta T = \frac{Q}{C_S \cdot A} = \frac{(I_{in} - I_{out})\Delta t}{C_S}$.

Dynamics of T

*1 $\quad I_{in} - I_{out} = \frac{1}{4} S(1-\alpha) - \varepsilon\sigma T^4$

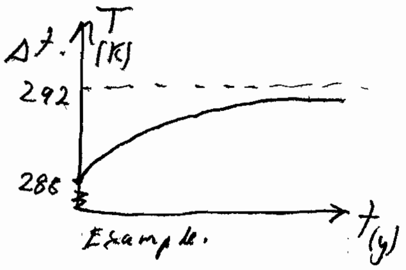
*2 $\quad \Delta T = \frac{1}{C_s}(I_{in} - I_{out}) \cdot \Delta t$

Given I_{in} and a temperature T, e.g. so $I_{in} > I_{out}$.

*1 gives $I_{in} - I_{out}$. *2 gives ΔT in timestep Δt.

New T = Old $T + \Delta T$.

*1 gives new $I_{in} - I_{out}$. *2 gives new ΔT, etc.



Examples of models of energy balance of Earth

See JordEne1.pdf

Model 1 No atmosphere.

Earth: $\frac{1}{4}(1-\alpha)S = \sigma T_E^4$ $\left[\frac{1}{4}S = 339\,Wm^{-2}\right]$. $T_E = 254\,K$.

Model 2 Atmosphere passes radiation from Universe to Earth, but absorbs radiation from Earth and reradiates to Earth & Universe, each of σT_A^4.

Atmosphere: $\sigma T_E^4 = 2\sigma T_A^4$
Earth: $\frac{1}{4}(1-\alpha)S + \sigma T_A^4 = \sigma T_E^4$

$T_A = 254\,K$
$T_E = 302\,K$

Model 3 Atmosphere absorbs 19% of radiation from Universe.

Atmosphere: $\frac{1}{4} \times 0.19 S + \sigma T_E^4 = 2\sigma T_A^4$
Earth: $\frac{1}{4}(1-\alpha-0.19)S + \sigma T_A^4 = \sigma T_E^4$

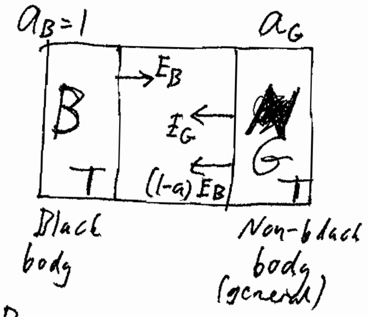
$T_A = 254\,K$
$T_E = 291\,K$

That atmospher absorbs all radiation from Earth is the worst considered "pure" greenhouse effect.
See model 4 for more realistic value at present, where atmosphere only from Earth absorbs $\sigma T_E^4 - 20\,Wm^{-2}$.

Compare also with diagram in Kirk p. 76 upper.

1. Kirchhoff's law of thermal radiation

Heat radiation from a body depends only on properties of the body, especially its temperature T, and does not depend on the surroundings. So the body emits radiation independently of the surroundings, but of course it also absorbs radiation from the surroundings.



Box in equilibrium. at temperature T

A black body absorbs all heat-rad.
Absorption coefficient: a.
Reflection coefficient: $1 - a = r$
For a blackbody: $a_B = 1$.
G has abs. coeff. $a_G < 1$.

B emits energy E_B. G emits energy E_G.
In thermodynamic equilibrium B emits the same energy (or power or intensity) as B receives.
B receives from G the energy emitted by G and the energy reflected from G: $E_G + (1 - a_G)E_B$

so: $$E_B = E_G + (1 - a_G)E_B \Leftrightarrow E_G = a_G E_B.$$

Emissivity is defined as $\varepsilon_G = \dfrac{E_G}{E_B}$. For B: $\varepsilon_B = 1$.

From $E_G = aE_B$ we get: $\varepsilon_G = a_G$ (at a given temp. T).
$\Rightarrow$ Kirchhoff's corrolar $\varepsilon_G < 1$ (because $a < 1$)

At a given temp. a body can never thermally radiate more energy than a blackbody at same temperature, when equilibrium

$\varepsilon_G = a_G = 1 - r_G$ >: "A poor reflector is a good emitter, a good reflector is a poor emitter."