Errors

From the Physics Guide (first exam. 2009):

Uncertainty and error in measurement			
1.2.6	Describe and give examples of random and systematic errors.	2	
1.2.7	Distinguish between precision and accuracy.	2	A measurement may have great precision yet may be inaccurate (for example, if the instrument has a zero offset error).
1.2.8	Explain how the effects of random errors may be reduced.	3	Students should be aware that systematic errors are not reduced by repeating readings.
1.2.9	Calculate quantities and results of calculations to the appropriate number of significant figures.	2	The number of significant figures should reflect the precision of the value or of the input data to a calculation. Only a simple rule is required: for multiplication and division, the number of significant digits in a result should not exceed that of the least precise value upon which it depends. The number of significant figures in any answer should reflect the number of significant figures in the given data.
Uncertainties in calculated results			
1.2.10	State uncertainties as absolute, fractional and percentage uncertainties.	1	
1.2.11	Determine the uncertainties in results.	3	A simple approximate method rather than root mean squared calculations is sufficient to determine maximum uncertainties. For functions such as addition and subtraction, absolute uncertainties may be added. For multiplication, division and powers, percentage uncertainties may be added. For other functions (for example, trigonometric functions), the mean, highest and lowest possible answers may be calculated to obtain the uncertainty range. If one uncertainty is much larger than others, the approximate uncertainty in the calculated result may be taken as due to that quantity alone.
Uncertainties in graphs **Aim 7:** This is an opportunity to show how spreadsheets are commonly used to calculate and draw error bars on graphs.			
1.2.12	Identify uncertainties as error bars in graphs.	2	

	Assessment statement	Obj	Teacher's notes
1.2.13	State random uncertainty as an uncertainty range (±) and represent it graphically as an "error bar".	1	Error bars need be considered only when the uncertainty in one or both of the plotted quantities is significant. Error bars will not be expected for trigonometric or logarithmic functions.
1.2.14	Determine the uncertainties in the gradient and intercepts of a straight-line graph.	3	Only a simple approach is needed. To determine the uncertainty in the gradient and intercept, error bars need only be added to the first and the last data points.

Systematic and random errors

See example in Excel-file errors.xls.

Rules for calculating uncertainties from random errors in results:

- If the expression of the result only contains sums and differences with factors:
  - Add all uncertainties to get the total absolute error.
  - Example: $y = 2a + 3b - 4c$. $\Delta y = 2\Delta a + 3\Delta b + 4\Delta c$.

- If the expression of the result only contains products and quotients with powers:
  - Add all uncertainties to get the total relative (percentage) error.
  - Example: $y = a^2 * b^3 / c^4$. $\Delta y/y = 2\Delta a/a + 3\Delta b/b + 4\Delta c/c$

Be aware of that these rules only applies to "pure" expressions of the forms mentioned above.
If you have a "mixed" expression you must find the total uncertainty from random errors by doing the "min-max-procedure" as shown in the example of the Excel-file errors.xls which is also reproduced as pdf-file below.

Latent heat of fusion - Melting ice - Error investigations

Constants (tables)

cw	ci	Lf
J/(kg ºC)	J/(kg ºC)	kJ/kg
4190	2218	334,4

Lf = cw*Mw/Mi (Tw - Tm) + ci Ti - cw Tm

Physical entity	Tw	Ti	Tm	Mw	Mi	Lf	Lf
Unit	ºC	ºC	ºC	g	g	J/kg	kJ/kg
Uncertainty of meas.	+/- 1 ºC	+/- 1 ºC	+/- 1 ºC	+/- 0,5 g	+/- 0,5 g		
Value	67	-9	15	26	13	352.948	353
High	68	-8	14	26,5	12,5	403.267	403
Low	66	-10	16	25,5	13,5	306.502	307
Middle value							355
Random error							48
Relative error %							14

Systematic errors

Ice not as cold	67	-5	15	26	13	361.820	362
Mixing temp. lower	67	-9	14	26	13	365.518	366
---	67	-9	15	26	13	352.948	353
---	67	-9	15	26	13	352.948	353

Deviation from table value

Physical entity	Lf low	Lf high	Lf Table	Lf Middle	Deviation	Deviation	
Unit	kJ/kg	kJ/kg	kJ/kg	kJ/kg		%	
	307	403	334	355	20	6,1	

Latent heat of fusion - Melting ice - Random errors

Random errors - precision of measurement
The random errors of measurements are judged as a +/- value, e.g. for temperature: +/- 1 ºC or for mass:+/- 0.5 g.

The random errors influences the calculated results. If for instance A and B are measured with random error +/- 0.5, then C = A - B will have a random error of +/- 1 g. The way to see this is:
How to create a high value? - in this case by a high A and a low B.
How to create a low value? - in this case by a low A and a high B.

Example: A = 26g, B = 39g, then C = A - B = 39 - 26 = 13g.
Assume random error on all measured masses: +/- 0.5 g. Then high and low values of C becomes:

High: C = 39.5 - 25.5 = 14g. Low: C = 38.5 - 26.5 = 12g.

Middle value: (High+Low)/2=(14+12)/2=13g. Random error: (High-Low)/2=(14-12)/2=1g.
Conclusion: C = (13 +/- 1)g.

Similar in a formular containing a fraction, x = A/B, to make x high you must make A high, B low, and to make x low you must make A low and B high.

In a formula like the one for Lf it is of course a more complicated problem. Let us look at the formula:

Lf = cw*Mw/Mi (Tw - Tm) + ci Ti - cw Tm

How to make Lf high? Well, Mw, Tw and Ti must be high, and Mi, Tm must be low.
Better check this yourself by looking at the formula of Lf - or by experimenting with different values in your table, making Excel show the effect on the result - see the Excel-part called "Error investigations".
Of course opposite with how to make Lf low.

As before you find the middle value of Lf = (high+low)/2 and the random error on Lf = (high-low)/2.
In the end present your result as: Lf = middle value +/- random error.
For example it could be: Lf = (358 +/- 65)kJ/kg, or better: Lf = (360 +/- 70)kJ/kg, now adjusting the number of significan figures better to the accuracy.
You can also decide to present the error as a percentage error: 70/360 = 0.194 = 20%.

It is then interesting to see if the tabled value is situated within the measured interval of Lf. In this example, where the table value is 334 kJ/kg we see this is the case. In this sense we can say, that our measurement confirmes the tabled value, but - we must say - with a rather high percentage error.

It is not always easy to judge from the formula what to do to make the result high or low. In some formulas the same entity plays different roles, e.g. being subtracted somewhere and multiplied elsewhere. But you can always check out the effect on the result of making the value of the entity high or low by choosing different values in your table and let Excel show the results.

The effect of random errors from different physical entities, like temperatures and masses, can be very different. Observe the effect of the proposed random errors in the ice-experiment. Which seems to create the strongest effect: the errors on temperatures or the errors on masses? The result of such an investigation can lead to a better judgement of the experiment - how much is it to trust? And the investigation can lead to ideas of doing more careful experiments - now you know which measurements are "critical".

Special rules for calculating total uncertainty from random errors in results

Example 1: **Expression: y = 2a + 3b – 4c** Expression only with sum and differences with factors.

		Special rule: Add up abs. errors				max-min-method:			
		+/-	Factors		fac*delta	max	min	Middle	abs.error
a	25	0,1	2		0,2	25,1	24,9	25	0,1
b	20	0,2	3		0,6	20,2	19,8	20	0,2
b	12	0,4	4		1,6	11,6	12,4	12	-0,4
Expression:	**62,0**		**Abs. err:**		**2,4**	**64,4**	**59,6**	**62,0**	**2,4**
Rel.error: No rule, but calculate by abs.error / value:					3,9%				

Example 2: **Expression: y = a^2*b^3/c^4** Expression only with products and quotients with powers.

		Special rule: Add up rel. errors				max-min-method:				
		+/-	Powers	rel.error	pow*rel.er.	max	min	Middle	abs.error	rel.error
a	25	0,1	2	0,004	0,008	25,1	24,9	25	0,1	0,004
b	20	0,2	3	0,01	0,03	20,2	19,8	20	0,2	0,01
b	12	0,4	4	0,033333	0,133333	11,6	12,4	12	-0,4	-0,033333
Expression:	**241**		**Rel. err:**		**17,1%**	**287**	**204**	**245**	**41,6**	**17,0%**
Abs.error: No rule, but calculate by rel.error * value:					41,3					41,6

Latent heat of fusion - Melting ice - Systematic errors

Systematic errors - accuracy of measurement

The systematic errors are caused by some known or unknown effects on the measurements which are not considered and included in the theory and formulas. It is typical that one specific systematic error pulls the measurements - and the results - in one direction, while random errors create a more random variation on measurements and results jumping up and down.

Example: You consider that the ice taken from the freezer probably does not have the original temperatur measured in the freezer, but higher, when dumped in the mixing water. So your value of Ti is probably too low. Could that be the reason for your resulting Lf to be higher than the table-value?

Now simply try to suggest a new - more realistic - value of Ti. Observe the effect on the resulting Lf. Does this error explain that our Lf became higher than the table value? The answer in this case is no, because we actually observe that a higher value of Ti creates an even higher value of Lf. This is not saying that this systematic error is not relevant. But there could be other errors pulling in positive or negative directions.

And we observe in this case something else, which is interesting. The effect of changing the value of Ti does not change the value of Lf very much - much less than the effects of the random errors (of the masses). This is of course not always the case, but in any case by this investigation, we get important information concerning the circumstances of our experiment.

Try to suggest other possible systematic errors - do not mix these with the random errors! - and see if you can investigate the possible effect of these errors on the resulting value of Lf.