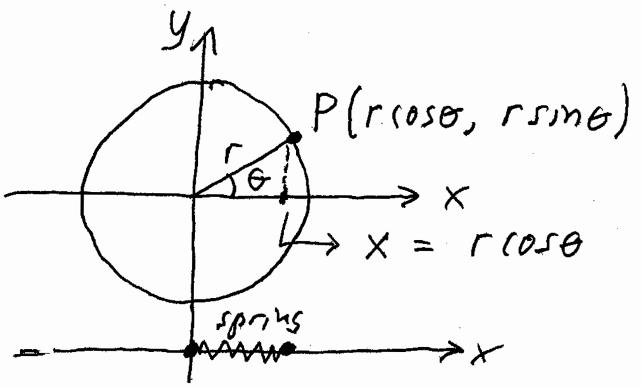
Harmonic oscillation (spring)



Remember uniform circular motion?

$\theta = \omega t$. $\omega = \frac{2\pi}{T}$

Now look at x-axis only (projection or "shadow")

$x = r\cos(\omega t)$. $v_x = -r\omega\sin(\omega t)$. $a_x = -r\omega^2\cos(\omega t)$

So always $a_x = -\omega^2 x$. and $F_x = -m\omega^2 x$.

Compare this with a spring (Hooke's law):

$F_x = -kx$ or $a_x = -\frac{k}{m}x$.

So the shadow from the uniform circular motion on the x-axis follows the movement of a mass on a spring following Hooke's Law. All the above equations are valid for the spring.

For a spring r is normally called A (amplitude).

T	Period of time [s]	
f	Frequency	$f = \frac{1}{T}$ or $T = \frac{1}{f}$.
ω	angular velocity [s^{-1}] or ang. frequency	$T = \frac{2\pi}{\omega}$ or $\omega = \frac{2\pi}{T}$
A	Amplitude	$\omega = 2\pi f$.

$a_x = -\omega^2 x = -\frac{k}{m}x$ $\omega = \sqrt{\frac{k}{m}}$. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

$x = A\cos(\omega t) = A\cos(2\pi f \cdot t)$. $x_{max} = A$

$v_x = -A\omega\sin(\omega t)$. $v_{x\,max} = A\omega$

$a_x = -A\omega^2\cos(\omega t) = -\omega^2 x$. $a_{x\,max} = A\omega^2$

Energy of harmonic oscillator (spring)

$$E_{kin} = \frac{1}{2} m v^2 = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t)$$

$$E_{pot} = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t), \quad \text{but } k = m\omega^2 \text{ so:}$$

$$E_{pot} = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t).$$

$$E_{tot} = E_{kin} + E_{pot} = \frac{1}{2} m \omega^2 A^2 \left(\sin^2(\omega t) + \cos^2(\omega t)\right) = 1$$

$$= \frac{1}{2} m \omega^2 A^2 \times 1 \quad \left(\text{or } = \frac{1}{2} k A^2\right)$$

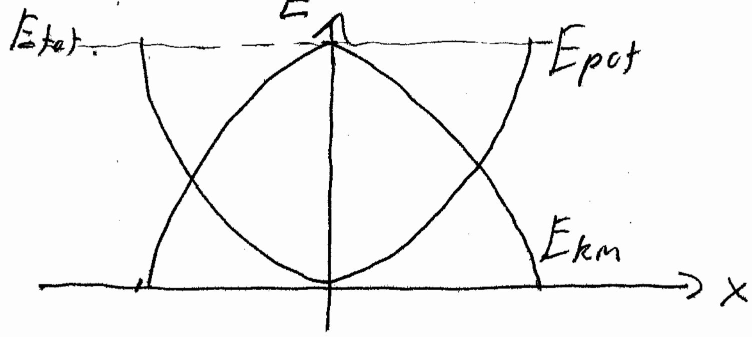
E_{tot} is constant (as expected!)

(Trigonometry: $\sin^2\theta + \cos^2\theta = 1$ always!)

Also: $\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} m \omega^2 A^2 = \frac{1}{2} k A^2$

or $\frac{1}{2} m v^2 + \frac{1}{2} m \omega^2 x^2 = \frac{1}{2} m \omega^2 A^2$

or $v^2 + \omega^2 x^2 = \omega^2 A^2 \; (= v_{max}^2)$



Energies as functions of x.

Se also: Giancoli ed. 6: 11-1 – 11-3

Kirk ed. 2: 4, p. 33–37